N.S.L. and Cons. of Energy

(L-14)

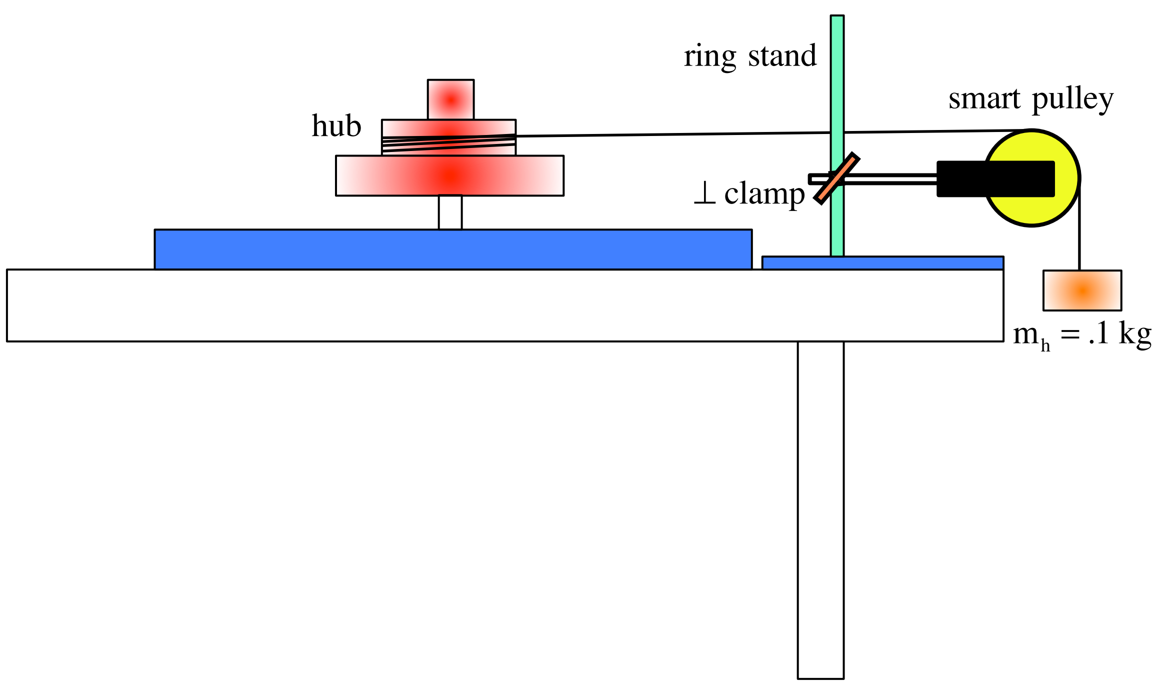
There is a close parallel between the theoretical dynamics of linearly translating objects and the dynamics of rotating objects. For instance, just as Newton's Second Law predicts that a *translational acceleration* must come as a consequence of a *net applied force*, the rotational analog maintains that a *rotational acceleration* must come as a consequence of a *net applied torque*. This lab is designed to see if the relationship between *torque* and *angular acceleration* is really as the theory predicts. It has the added delight of requiring you to use *conservation of energy* to determine the *moment of inertia* of the rotating system. Yippee!!!

PROCEDURE—DATA

Part A: (general set-up and procedure)

a.) Plug a Smart Pulley into your Lab Pro, turn on your computer and open up the *Velocity versus Time* program found on the desktop.

b.) Use a ring stand and right-angle clamp to mount the Smart Pulley. The string should be wrapped around the second tier of the mounted hub and should be oriented in the horizontally. Use a 100 gram (.1 kg) hanging mass. The set-up is shown below. Construct it.



c.) Use a caliper to measure the diameter of the hub around which the string is wound.

Part B: (data)

d.) Allow the hanging mass to free fall while the Logger Pro program is running (be sure the string isn't rubbing on anything as it unwinds about the hub, and stop the falling mass before it hits the floor). This will generate a *velocity versus time* graph. Highlight, then magnify the linear region of that graph. Once done, use the REGRESSION LINE tool to determine the slope of that line (and the acceleration of the motion). Print the graph (not the page, just the graph).

e.) Remove the REGRESSION LINE data, then click on the label "velocity" found on the vertical axis of the graph. A dialogue box will appear. In that box, click on "distance." That will yield a *position versus time* graph. Highlight the parabolic part of the graph, magnify it, then print that graph.

CALCULATIONS

Part A: (torque from rxF)

1.) Preface: In this section, we want to determine the torque being provided to the hub system by using the vector cross product *rxF*, where *F* is the net force acting on the hub (in this case, this is the tension *T* in the string—we are assuming no friction) and *r* is the distance between the *axis of rotation* and the place where *T* acts on the hub. To carry this cross product out, we need *T*:

To get *T*:

a.) Draw a f.b.d. for the forces acting on the hanging mass, then use N.S.L. to derive a general algebraic expression for the tension *T* in the string (remember, the graph gave you the *acceleration* of the hanging mass).

b.) Use the relationship derived in *Part a* and the numbers taken during lab to determine a numeric value for the tension in the line *T* during the free fall.

2.) Use *rxF* to derive an expression for the net torque acting on the rotating system (you might want to draw a new f.b.d. for the forces acting on the hub, just to be sure you see what's happening). Ignore friction and call this calculated value Γ_{rxF}.

Part B: (torque from $I\alpha$)

3.) The rotational counterpart to N.S.L. states that the net torque acting on a system must be proportional to the system's angular acceleration α with the proportionality constant being *the moment of inertia I*. That is, the net torque must equal $I\alpha$. To get *I* and α:

a.) Use $a = R\alpha$ to determine the *angular acceleration* of the rotating hub as the hanging mass fell.

b.) Use the *conservation of energy* and the assumption that the system was essentially frictionless to derive a general algebraic expression for the *moment of inertia* of the disk. This should be in terms of the hanging mass m_h, a distance *d* the hanging mass dropped between times t_1 and t_2 (your choice as to those times, depending upon your graphs), *g*, the hanging mass's velocities v_1 and v_2 (i.e., the velocities at times t_1 and t_2) and the hub's radius *r*.

c.) Use the relationship derived in *3b* to determine a numeric value for the hub's *moment of inertia*.

4.) Determine the net torque acting on the system using the vector product $I\alpha$. Call this $\Gamma_{I\alpha}$.

5.) We are now in a position to decide whether the rotational analog to N.S.L. really works (i.e., if $(rxF)_{net} = I\alpha$).

a.) Do a % comparison between Γ_{rxF} and $\Gamma_{I\alpha}$.

b.) Did the deviation seem big? If so, explain from whence the discrepancy is most likely to have come (and if not, say HALLELUJAH BROTHER).